Gr 12 June Paper 2 150 2015

1.1. 1. $\bar{x} - M$

$= 95 - 85$

$= 10$

> 0

$\therefore$ distances are positively skewed (skewed to the right)

2

1.1. 2. $Q_1 = 69$ $IQR = 113 - 69$

$= 44$

$LF = Q_1 - 1{,}5 \cdot IQR$

$= 69 - 1{,}5 \cdot 44$

$= 3$

Now, $2 < 3$

$\therefore$ 2 is an outlier

3

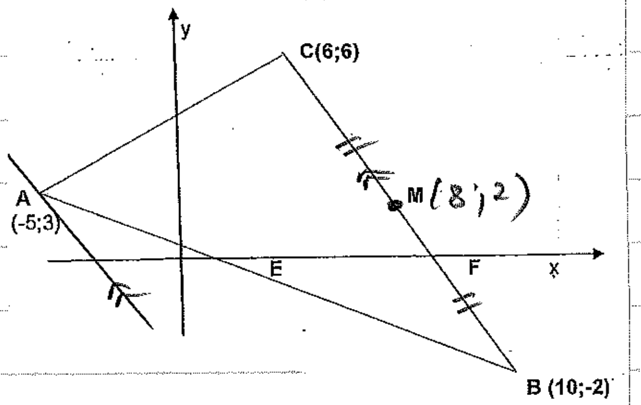
1.2.

a	b	c	d	e	f	g
1	2	3	4	5	6	7
min	Q_1		M		Q_3	max
						42 g
42−35 7 a			d 23			
	37−22 15 b				23+14 37 f	
		15+6 21 c				

$\frac{7+15+21+23+e+37+42}{7} = 25$

$\therefore e = 30$ e $25 \times 7 - 145$

10



3.1. $m_{BC} = \frac{-2-6}{10-6} = -2$

2

3.2. $BC = \sqrt{(-2-6)^2 + (10-6)^2}$

$= \sqrt{80}$

$= \sqrt{16 \cdot 5}$ NB!!

$= 4\sqrt{5}$

4

3.3. $x_M = \frac{10+6}{2}$ $y_M = \frac{-2+6}{2}$

$= 8$ $= 2$

$\therefore M(8;2)$

2

3.4. $m_{AD} = -2$ $AD \parallel BC$

$\therefore y = -2x + c$

Sub $A(-5;3)$

$3 = -2(-5) + c$

$-7 = c$

$\therefore y = -2x - 7$

Diagram Sheet A

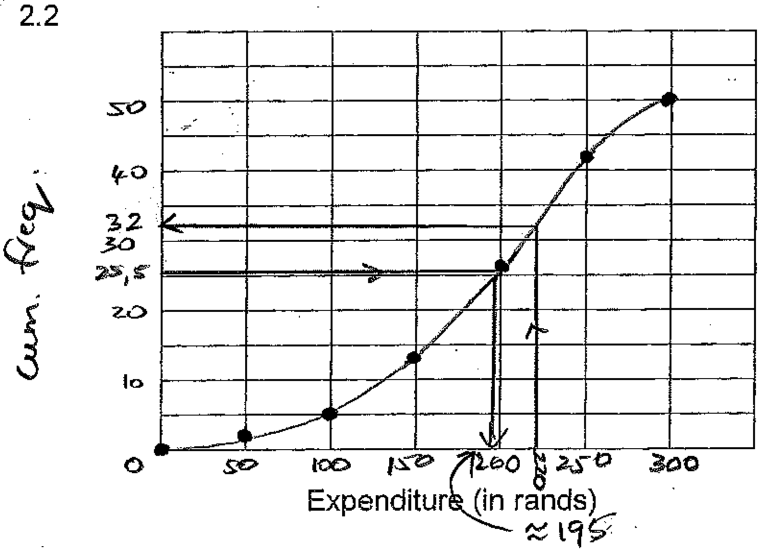
Name: ____________________

Question 2.1

Expenditure (rands)	Number of Matrics	Cumulative frequency
$0 \leq x \leq 50$	2	2
$50 < x \leq 100$	$a = 3$	5
$100 < x \leq 150$	8	13
$150 < x \leq 200$	13	26
$200 < x \leq 250$	16	$b = 42$
$250 < x < 300$	8	50

2

2.2



✓ grounding point (0;0)
✓ upper end of interval
✓ plotting
✓ smooth curve

4

2.3 $T_1 - T_{50}$ $M = T_{\frac{1}{2}(50+1)} = T_{25,5} \approx R195$

2

2.4 $\leq R220 = 32$

$\therefore > R220 = 50 - 32$

$= 18$ learners

2

3.4. $y + 2x + 7 = 0$

3

3.5. $M(8;2)$ $C(6;6)$

$(x-8)^2 + (y-2)^2 = r^2$

sub $C(6;6)$ ... or B

$(6-8)^2 + (6-2)^2 = r^2$

$20 = r^2$

$\therefore (x-8)^2 + (y-2)^2 = 20$

3

3.6. $A(-5;3)$ $M(8;2)$

$r = \sqrt{20}$

$AM = \sqrt{(2-3)^2 + (8-(-5))^2}$

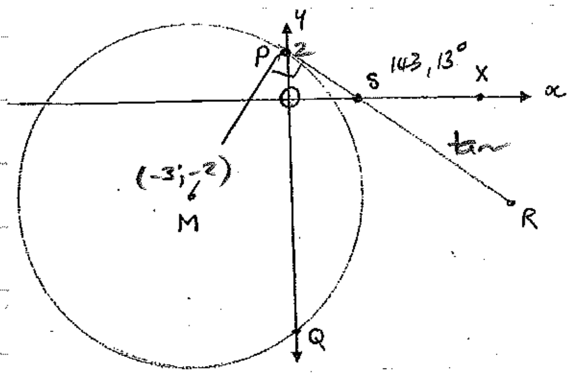
$= \sqrt{170}$

$AM > r$

$\therefore$ A lies outside ⊙

3

4.



4.1. 1. $x^2+6x+(+3)^2+y^2+4y+(+2)^2 = 12+9+4$

$(x+3)^2 + (y+2)^2 = 25$

$\therefore M(-3;-2)$

5

2. $A = \pi r^2$

$= \pi . 25$

$= 78,54\ u^2$

1

3. yint: $y^2+4y-12=0$

$x=0$ $(y-2)(y+6)=0$

$\therefore y = 2$ or $\cancel{-6}$

$\therefore P(0;2)$

4

4. $M(-3;-2)$ $P(0;2)$

$m_{MP} = \frac{2-(-2)}{0-(-3)} = \frac{4}{3}$

$\therefore m_{tan} = -\frac{3}{4}$ tan ⊥ rad

$\therefore y = -\frac{3}{4}x + 2$

3

4.1.5. $\tan\theta = m$

$\tan P\hat{S}X = -\frac{3}{4}$ ✓

ref ^ $= 36,86\ldots°$

tan − an

Q II : $P\hat{S}X = 143,13°$ ✓

2

✓R

6. $R\hat{P}O + 90° = 143,13°$ Ext ^Δ

$R\hat{P}O = 53,13°$

✓

2

4.2. $(x+3)^2 + (y+2)^2 = 25$

$M(-3;-2)$ $r = 5$

- reflect $y=0$ (x axis)

$(x+3)^2 + (-y+2)^2 = 25$

- $r_{new} = \frac{5}{2}$

$(x+3)^2 + (-y+2)^2 = \left(\frac{5}{2}\right)^2$

✓ ✓ $= \frac{25}{4}$ ✓

3

5. $\sin(A+B)$

$= \cos(90° - (A+B))$ ✓

$= \cos(90° - A - B)$

$= \cos((90°-A) - (B))$ ✓

✓ $= \cos(90°-A)\cos B + \sin(90°-A)\sin B$

$= \sin A\cos B + \cos A\sin B$

3

6. WOC

6.1. $x = 3\sin\theta$ $y = 3\cos\theta$

$x^2 + y^2$

$= (3\sin\theta)^2 + (3\cos\theta)^2$

$= 9\sin^2\theta + 9\cos^2\theta$ ✓

$= 9(\sin^2\theta + \cos^2\theta)$ ✓

$= 9 \cdot 1$

$= 9$ ✓

3

6.2. • $\sin(-x) = -\sin x$

• $\tan(-x+180°) = \tan(180°-x)$

$= -\tan x$

• $\cos(1260°+x) = \cos(180°+x)$

$= -\cos x$

• $\sin(90°+x) = \cos x$

• $\cos(-x-180°) = \cos(-x+180°)$

$= \cos(180°-x)$

$= -\cos x$

∴ we get

$(-\sin x)(-\tan x)(-\cos x) + (\cos x)(-\cos x)$ ✓✓✓✓✓

$= (-\sin x)\left(-\frac{\sin x}{\cos x}\right)(-\cos x) + (\cos x)(-\cos x)$ ✓

$= -\sin^2 x - \cos^2 x$

$= -(\sin^2 x + \cos^2 x)$

$= -1$ ✓

7

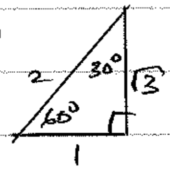
6.3. $\bullet\ \sin 150^\circ$

$= \sin(180^\circ - 30^\circ)$

$= \sin 30^\circ$

$= \frac{1}{2}$

NB



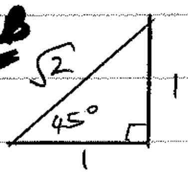
$\bullet\ \cos 225^\circ$

$= \cos(180^\circ + 45^\circ)$

$= -\cos 45^\circ$

$= -\frac{1}{\sqrt{2}}$

NB



$\bullet\ \sin 260^\circ$

$= \sin(270^\circ - 10^\circ)$

$= -\cos 10^\circ$

$\bullet\ \cos(-350^\circ)$

$= \cos 10^\circ$

$\bullet\ \sin 315^\circ$

$= \sin(360^\circ - 45^\circ)$

$= -\sin 45^\circ$ $\quad\Delta$ above

$= -\frac{1}{\sqrt{2}}$

$\therefore$ we get

$$\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{\sqrt{2}}\right)\left(-\cos 10^\circ\right)}{\left(\cos 10^\circ\right)\left(-\frac{1}{\sqrt{2}}\right)}$$

$= -\frac{1}{2}$

6

7.1. 1. LHS

$$= \frac{2\tan x - \sin 2x}{2\sin^2 x}$$

$$= \frac{\frac{2\sin x}{\cos x} - 2\sin x\cos x}{2\sin^2 x}$$

$$= \frac{\frac{2\sin x - 2\sin x\cos^2 x}{\cos x}}{2\sin^2 x}$$

$$= \frac{2\sin x(1-\cos^2 x)}{\cos x} \times \frac{1}{2\sin^2 x}$$

$$= \frac{2\sin x \cdot \sin^2 x}{2\cos x \cdot \sin^2 x}$$

$$= \frac{\sin x}{\cos x}$$

$= \tan x$

$=$ RHS

5

2. ID is UD when

$\bullet\ \tan x = \text{UD}$

$\frac{\sin x}{\cos x} = \text{UD}$

$\cos x = 0$

$x = 90^\circ + k180^\circ$

$2\sin^2 x = 0$

$\sin x = 0$

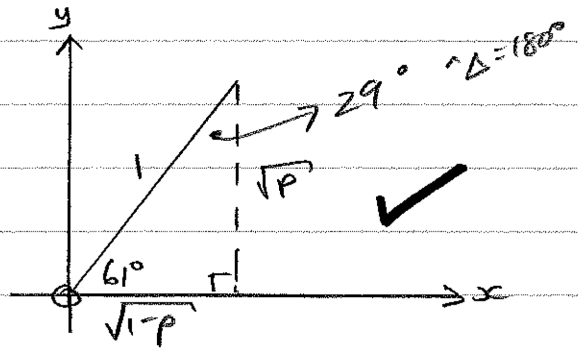
$x = k.180^\circ$

$(k \in \mathbb{Z})$

3

7.2. 1. $\sin 61° = \sqrt{p}$

$= \frac{\sqrt{p}}{1} \quad \frac{y}{r}$



$x^2 + (\sqrt{p})^2 = 1^2$ Pythag

$x^2 = 1 - p$

$x = \pm\sqrt{1-p}$

$= \sqrt{1-p}$ reject −

$\cos 61° = \frac{x}{r}$

$= \frac{\sqrt{1-p}}{1}$

$= \sqrt{1-p}$ **2**

2. $\sin 241°$

$= \sin(180° + 61°)$

$= -\sin 61°$

$= -\sqrt{p}$ **2**

3. $\cos 122°$

$= \cos(2 . 61°)$

$= 1 - 2\sin^2 61°$

$= 1 - 2(\sqrt{p})^2$

$= 1 - 2p$ **3**

7.2. 4. $\cos 73° \cos 15° + \sin 73° \sin 15°$

$= \cos(73° - 15°)$

$= \cos 58°$

$= \cos(2 . 29°)$

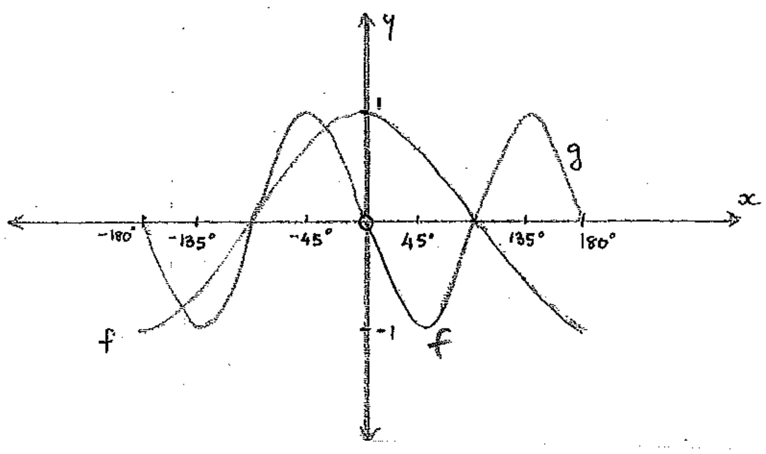
$= 2\cos^2 29° - 1$

$= 2\left(\frac{a}{h}\right)^2 - 1$

$= 2\left(\frac{\sqrt{p}}{1}\right)^2 - 1$

$= 2p - 1$ **4**

8.



$f: y = \cos x \quad g: y = -\sin 2x$

8.1. $f(x) = g(x)$

$\cos x = -\sin 2x$

$\cos x = -2\sin x \cos x$

$\cos x + 2\sin x \cos x = 0$ ✓

$\cos x(1 + 2\sin x) = 0$ ✓

$\cos x = 0$ or ✓ $\sin x = -\frac{1}{2}$

$x = 90° + k180°$ ✓ $\quad$ ref $\angle = 30°$

$\sin - $ ve

✓ QIII : $x = 210° + k360°$

QIV : $x = 330° + k360°$

$(k \in \mathbb{Z})$

- x ; $-90°$; $90°$; x
- x ; $-150°$; $\cancel{210°}$; x
- x ; $-30°$; $\cancel{330°}$; x

✓

$\therefore x = -150°; -90°; -30°$ or $90°$

6

OR

$\cos x = -\sin 2x$

$\cos(90° + 2x) \quad \cos(270° - 2x)$

II $\qquad$ III

$(k \in \mathbb{Z})$ ✓

$\cos x = \cos(90° + 2x)$ ✓ or $\cos x = \cos(270° - 2x)$ ✓

$x = 90° + 2x + k360° \qquad x = 270° - 2x + k360°$

$-x = 90° + k360° \qquad 3x = 270° + k360°$

$x = -90° + k360°$ ✓ $\qquad x = 90° + k120°$ ✓

- x ; $-90°$; x
- x ; $-150°$; $-30°$; $90°$; x

✓

$\therefore x = -150°; -90°; -30°$ or $90°$

6

8.2. 1. $f(x) > g(x)$

$y_f > y_g$

$x \in [0°; 90°)$ ✓✓ **2**

2. $f(x) \cdot g(x) \leq 0$

$y_f \cdot y_g \leq 0 \quad \bar{0}$

$\therefore x = -180°$ or $-90°$ or $x \in [0°; 180°]$

✓ ✓ ✓ **3**

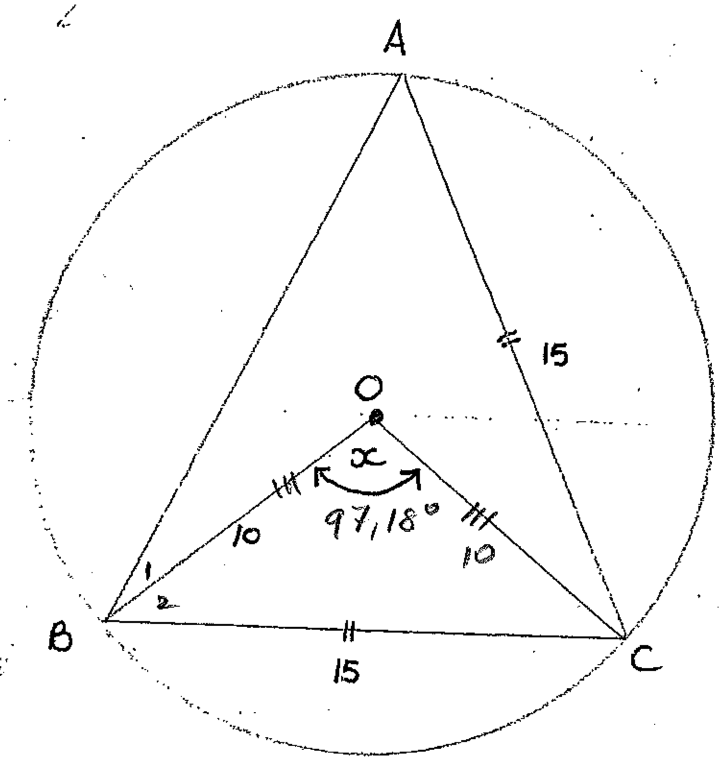
3. $h(x) = \cos(x + 20°)$ ✓ **1**

Diagram Sheet B

Name: ______________________

Question 9

9.



9.1	$OB = OC = 10$ radii	ref ∠ = 82,81...°
	$15^2 = 10^2 + 10^2 - 2(10)(10)\cos x$	cos − in
	$25 = -200\cos x$	Q II : $x = 97,18°$
	$\frac{-1}{8} = \cos x$	

3

(3)

9.2

$\hat{A} = 48,59°$ ∠@ centre = 2 ∠@ ⊙'ce SR

$\therefore \hat{B}_{1+2} = 48,59°$ isos Δ, sides = SR

$\therefore A\hat{C}B = 82,82°$ ∠'s Δ = 180° SR

3

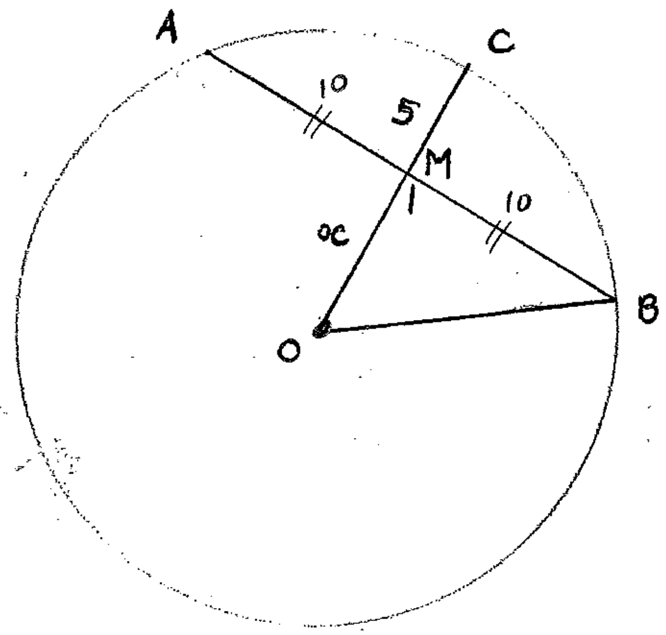
(3)

Diagram Sheet C

Name: ______________________________

Question 10

$AB = 20$



10.1 $MB = 10$ units (1) 1

10.2 line centre ⊙ to midpt chord is $\perp$ to chord (1) 1

10.3 $OC = x + 5$ (1) 1

10.4 $OB = x + 5$ radii

$x^2 + 10^2 = (x+5)^2$ S Pythag R

$x^2 + 100 = x^2 + 10x + 25$

$75 = 10x$

$7{,}5 = x$ 3

(3)

[6]

Diagram Sheet D

Name: ______________________________

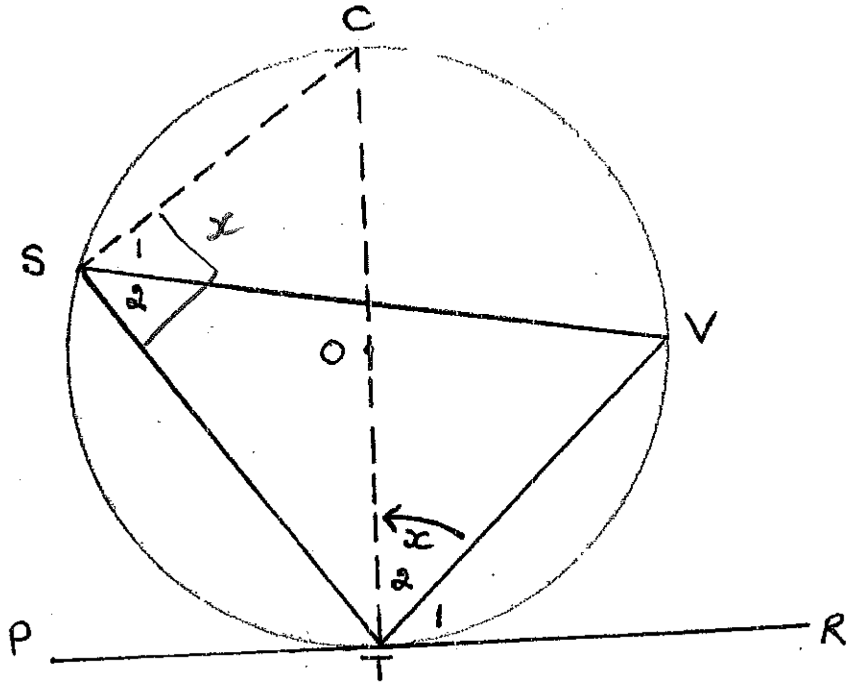
Question 11

11.1 Complete the statement of the following theorem:

The exterior angle of a cyclic quadrilateral is equal to ....

the opposite interior angle ✓ (1) 1

11.2 In the diagram below, the circle with centre O passes through points S, T and V. PR is a tangent to the circle at T. VS, ST and VT are joined. **Construction: Draw diameter TC and join CV.** Let VTC = $T_2 = x$

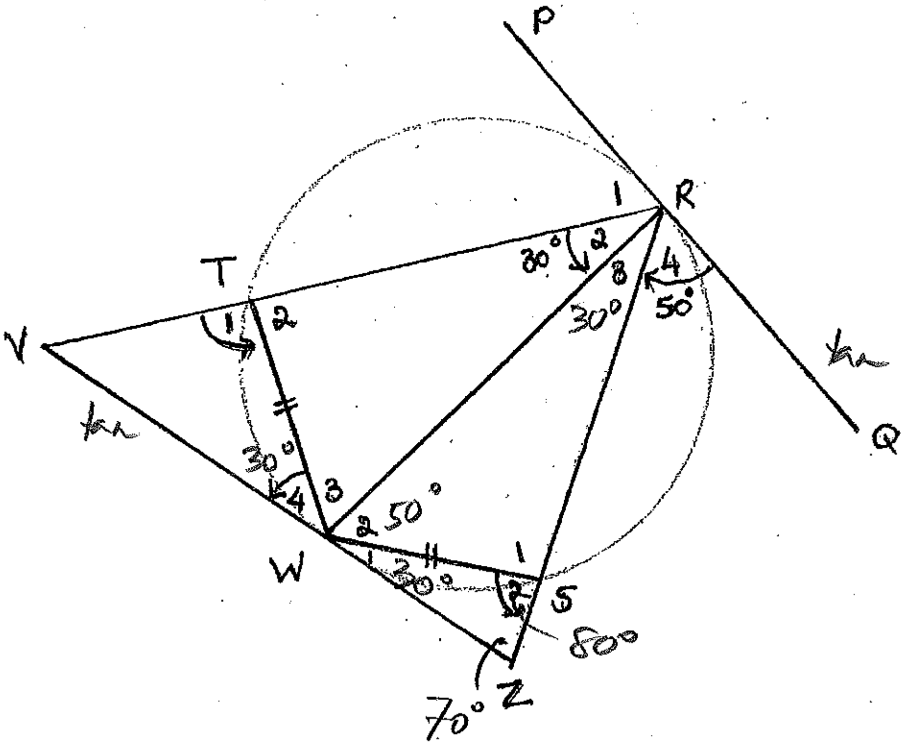


Statement	Reason
$S_1 + S_2 =$ 90° ✓ S	∠ in semi ⊙ = 90° ✓ R
$T_1 + T_2 =$ 90° ✓ S	tan ⊥ rad ✓ R
$T_2 = S_1 = x$	∠s in same ⊙ segm = ✓ R
$\therefore T_1 = 90° - x$	
and $S_2 = 90° - x$	
$\therefore$ VTR = VST	

5

Diagram Sheet E

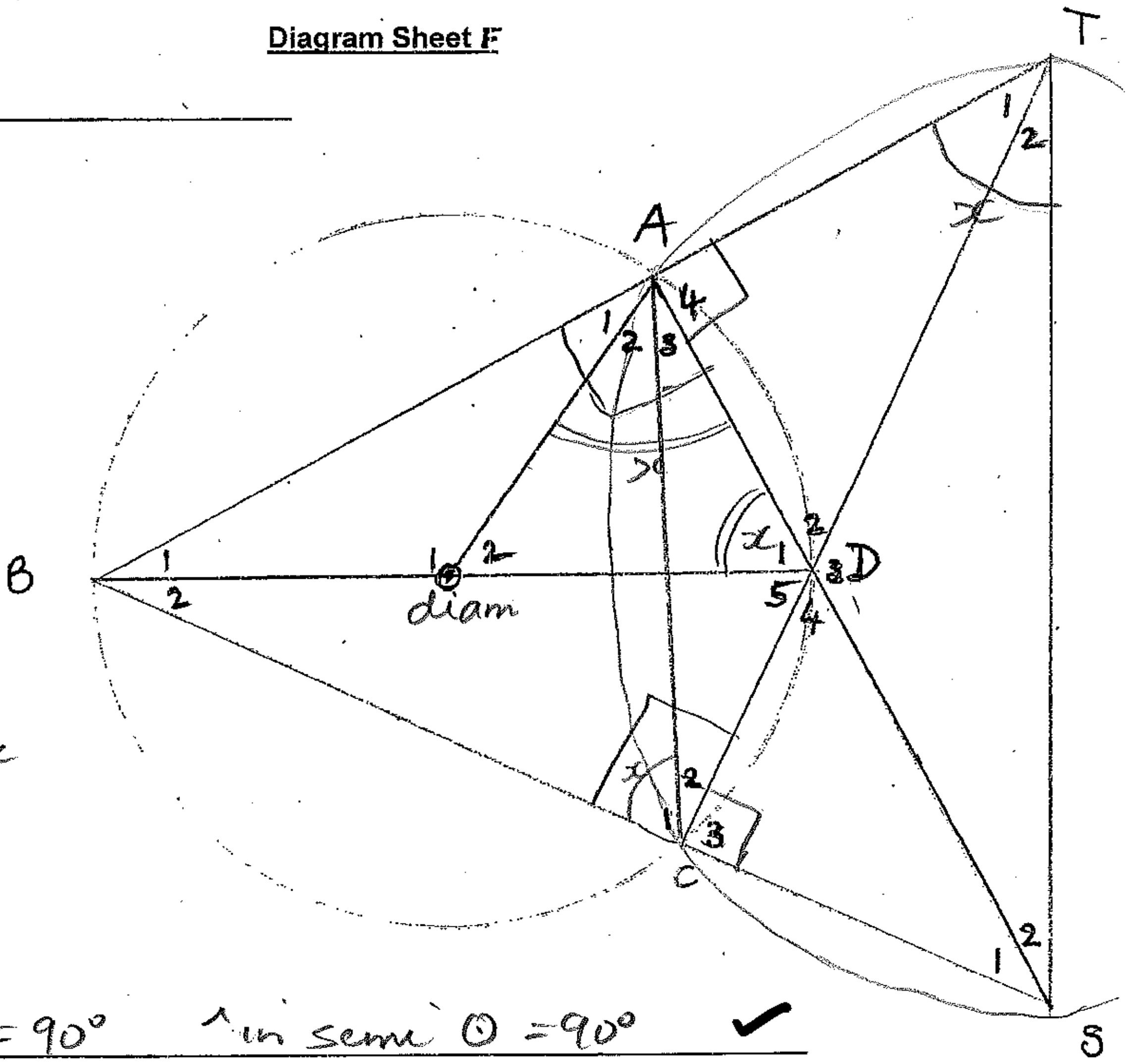
Name: ______________________________



11.3.1	= chords ✓	(1)	1
11.3.2	$\hat{W}_4 = 30°$ ✓ ∠ tan chord ✓		3
	$\hat{W}_1 = 30°$ ✓ ∠ tan chord	(3)	
11.3.3.1	$W_2 = 50°$ ✓S ∠ tan chord ✓R		3
	∴ $\hat{S}_2 = 80°$ ✓SR Ext ∠ Δ	(3)	
11.3.3.2	$\hat{Z} = 70°$ ✓SR ∠'s Δ = 180°		2
	$\hat{V} = 50°$ ✓SR ∠'s Δ = 180°	(2)	
		[14]	

(11)

Name: ____________________________



Let $\hat{D}_1 = x$

12.1 $\hat{C}_{1+2} = 90°$ ∠ in semi ⊙ = 90°

∴ $\hat{C}_3 = 90°$ ∠'s str line = 180°

Sim, $\hat{A}_4 = 90°$

∴ $\hat{C}_3 = \hat{A}_4$ both = 90°

∴ ATSC is cq conv ∠'s in same ⊙ segm = (4) 4

12.2 $\hat{C}_1 = x$ ∠'s in same ⊙ segm =

∴ $\hat{T}_{1+2} = x$ ext ∠ cyclic quad

∴ $\hat{D}_1 = \hat{T}_{1+2}$ both = x (3) 3

ie $A\hat{D}B = A\hat{T}S$

12.3 $\hat{A}_{2+3} = x$ isos Δ, sides =, radii OR $\hat{A}_2 = \hat{T}_1$

∴ $\hat{T}_{1+2} = \hat{A}_{2+3}$ both = 90°

∴ OA is tangent conv ∠ tan chord

(4) 4

[11]